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Comparison of Emittance Tuning Simulations in the NLC and TESLA Damping Rings

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Abstract

Vertical emittance is a critical issue for future linear collider damping rings. Both NLC and TESLA specify vertical emittance of the order of a few picometers, below values currently achieved in any storage ring. Simulations show that algorithms based on correcting the closed orbit and the vertical dispersion can be effective in reducing the vertical emittance to the required levels, in the presence of a limited subset of alignment errors.

1 Introduction

The specified normalized extracted emittance for both the TESLA damping rings [1] and the NLC main damping rings [2] is 0.02 µm rad. The required equilibrium emittance can be found from:

$$\gamma\varepsilon(t) = \gamma\varepsilon_{\mathrm{inj}} \mathrm{e}^{-2t/\tau} + \gamma\varepsilon_{\mathrm{equ}} \left(1 - \mathrm{e}^{-2t/\tau}\right)$$

where $\gamma\varepsilon_{\mathrm{inj}}$ is the normalized injected emittance, $\gamma\varepsilon_{\mathrm{equ}}$ is the normalized equilibrium emittance, t is the time after injection, and τ the damping time. Relevant parameters are given in Table 1. The vertical emittance is more than two orders of magnitude smaller than the horizontal, and expected to be much more difficult to achieve; in this note, we therefore focus entirely on the vertical emittance. The NLC requires a flat beam only in the main damping rings. The positron pre-damping ring may be operated with a fully coupled beam, and we therefore consider only the main damping rings here.

Table 1

Parameters determining required equilibrium vertical emittance in TESLA and NLC damping rings.

	TESLA e^+ Ring	NLC MDR
Energy	5.0 GeV	1.98 GeV
Relativistic Factor	9785	3875
Store Time	200 ms	25 ms
Damping Time	28.0 ms	5.00 ms
Normalized Injected Emittance	0.01 m rad	150 µm rad
Normalized Extracted Emittance	0.02 µm rad	0.02 µm rad
Normalized Equilibrium Emittance	0.0138 µm rad	0.0132 µm rad
Geometric Equilibrium Emittance	1.41 pm rad	3.40 pm rad

Vertical emittance is generated by betatron coupling and vertical dispersion in regions where there is synchrotron radiation, and by the non-zero vertical opening angle of the radiation. The opening angle of the radiation places a lower limit on the vertical emittance [3]:

$$\varepsilon_{y,\min} = \frac{13}{55}\frac{C_q}{J_y}\frac{\oint \beta_y / |\rho^3| \mathrm{d}s}{\oint 1/\rho^2 \mathrm{d}s}$$

For the TESLA positron ring this evaluates to 0.11 pm rad, and for the NLC MDR we find 0.24 pm rad; these values are less than 10% of the required equilibrium emittance in each case.

To achieve the desired vertical emittance, both the vertical dispersion and betatron coupling must be corrected to within some limits. Vertical dispersion is generated by vertical steering, and by coupling from the horizontal plane, for example in a vertically offset sextupole. Betatron coupling is generated principally from skew quadrupole fields, either from normal quadrupoles rotated about the beam axis, or from vertically offset sextupoles. The vertical emittance is therefore primarily an alignment issue. Optical errors will also play a role, however, by affecting the response matrices used to achieve the correction.

In this note, we estimate the sensitivity of the present TESLA and NLC damping ring lattices to various misalignments, and describe algorithms proposed to achieve the required equilibrium vertical emittance. Finally, we present results of simulations showing the effectiveness of the correction procedures, and comment on the needs for further studies.

It is not our intention here to survey the achieved emittance in operating machines, although we feel strongly that it is important to use the experience of existing storage rings, and future work should be directed to practical application rather than limited to simulation. Here, we simply note that there are indications from several electron storage rings that vertical emittances below 20 pm (in some cases below 10 pm) have been achieved [4-8].

2 Sensitivity Indicators and Estimates

In the parameter regime of the damping rings, it is important to address the contributions to the vertical emittance from both the vertical dispersion and the betatron coupling. In the TESLA damping ring, the wiggler dominates the vertical emittance form spurious dispersion, since this is where 90% of the radiation energy loss occurs. In the NLC damping ring, the wiggler accounts for about 60% of the energy loss, and vertical dispersion in the arcs is also important.

An estimate of the vertical emittance generated by vertical dispersion is given by [9]:

$$\varepsilon_y = 2J_\varepsilon \frac{\langle \eta_y^2 \rangle}{\langle \beta_y \rangle} \sigma_\delta^2$$

The vertical emittance in the TESLA damping ring is a little more sensitive to the dispersion than the NLC, since the energy spread is larger, while the lattice functions are comparable. Neglecting the effects of betatron coupling, we find that the vertical dispersion in the TESLA wiggler must be corrected below 1.5 mm rms, while in the NLC damping ring lattice, the vertical dispersion must be below 3.5 mm rms (see Table 2). For comparison, to achieve a vertical emittance below 5 pm in the ATF prototype damping ring requires dispersion correction better than 4.3 mm rms. In practice, in both NLC and TESLA damping rings we find that the dispersion must be corrected to better than 1 mm rms, since there is a significant contribution from betatron coupling. Given the above limits, however, we can estimate the quadrupole rotations and sextupole misalignments that generate the appropriate amount of vertical dispersion. These should not be interpreted as tolerances, but merely as indicating the sensitivity of the dispersion to particular misalignments.

Table 2

Estimated dependence of vertical emittance on vertical dispersion in three damping ring lattices

Lattice and region of energy loss	$\varepsilon_y / \langle \eta_y^2 \rangle$
ATF arcs	2.7×10^{-7} m^{-1}
TESLA wiggler	5.6×10^{-7} m^{-1}
NLC full lattice	4.6×10^{-7} m^{-1}

For uncorrelated misalignments, the following relationships are readily derived from the standard expressions for the vertical closed orbit and vertical dispersion:

$$\langle y_{\mathrm{co}}^2 \rangle \approx \frac{\langle \beta_y \rangle}{8\sin^2 \pi\nu_y} \left(\sum_{\mathrm{quadrupoles}} \beta_{\mathrm{y}} (k_1 l)^2 \right) \langle \Delta Y_{\mathrm{quadrupole}}^2 \rangle$$

$$\langle \eta_y^2 \rangle \approx \frac{\langle \beta_y \rangle}{2\sin^2 \pi\nu_y} \left(\sum_{\mathrm{quadrupoles}} \beta_{\mathrm{y}} (k_1 l \eta_x)^2 \right) \langle \Delta\Theta_{\mathrm{quadrupole}}^2 \rangle$$

$$\langle \eta_y^2 \rangle \approx \frac{\langle \beta_y \rangle}{8\sin^2 \pi\nu_y} \left(\sum_{\mathrm{quadrupoles}} \beta_{\mathrm{y}} (k_2 l \eta_x)^2 \right) \langle \Delta Y_{\mathrm{sextupole}}^2 \rangle$$

ΔY represents a vertical misalignment, and $\Delta\Theta$ represents a rotation about the beam axis. In our simulations, the orbit and dispersion are recorded at the BPMs, and the relevant mean beta function in the above expressions is therefore the mean beta function at the BPMs. Relevant parameters are given in Table 3, and a comparison between analytic estimates of the sensitivity parameters and results from simulations are given in Table 4. Plots of some of the results are also given. Simulations have been carried out in MAD and MERLIN for TESLA, and in MERLIN for the NLC damping rings; results from MAD and MERLIN are in good agreement.

The TESLA damping ring will use coupling bumps in the straight sections to increase the vertical beam size, and reduce space-charge effects. Our studies so far have been limited to effects primarily in the vertical plane, and the correction algorithm has been constructed accordingly. Our results are significant therefore only for the lattice *without* the coupling bumps, and the correction algorithm will need to be extended to include horizontal correction in the case of the lattice with the coupling bumps. The TESLA damping ring also includes vertical steering, for the straight sections to follow the curvature of the Earth. This steering generates a small amount of dispersion that makes a negligible contribution to the vertical emittance, and is included in the simulations.

There is generally good agreement between the simulations and the analytic estimates of sensitivity to the various misalignments; this gives us some confidence in the simulation results. Although there appear to be differences between the rings, the results of the tuning simulations will be more significant. In particular, we find that it is relatively straightforward to correct the dispersion in the simulations of the TESLA and NLC damping rings, to the level that it makes a contribution of the order 0.1 pm to the vertical emittance. At this stage, we would suggest that the TELSA and NLC damping rings would operate in broadly the same regime as the ATF.

Table 3

Sensitivity Parameters

		ATF	TESLA e^+ Ring	NLC MDR
Vertical Tune	ν_y	8.7589	41.1915	11.1357
Mean Beta Function at BPMs	$\langle \beta_y \rangle$	4.6 m	12 m	7.1 m
Quadrupole Orbit Factor	$\sum_{\text{quadrupoles}} \beta_y (k_1 l)^2$	338 m^{-1}	563 m^{-1}	507 m^{-1}
Quadrupole Dispersion Factor	$\sum_{\text{quadrupoles}} \beta_y (k_1 l \eta_x)^2$	2.88 m	82.6 m	2.42 m
Sextupole Dispersion Factor	$\sum_{\text{sextupoles}} \beta_y (k_2 l \eta_x)^2$	4860 m^{-1}	4250 m^{-1}	1300 m^{-1}

The values in Table 3 and Table 4 should be used only as indicating the general behavior of the lattices under consideration. The quadrupole vertical alignment, for example, is of limited direct significance for the vertical emittance, since although the closed orbit distortion results in a beam offset in the sextupoles, the uncorrected closed orbit is typically dominated by the principal betatron modes, and the beam offset in the sextupoles is correlated around the ring as a result. Thus, the formulae given above should not be used to estimate the resulting vertical dispersion or vertical emittance in this case.

Table 4

Comparison of analytic estimates of alignment sensitivities, and results of simulations. Note that for TESLA, we consider the dispersion in the wiggler only; for NLC, we include the dispersion throughout the full lattice.

		TESLA e^+ Ring		NLC MDR	
		Analytic	Simulation	Analytic	Simulation
Vertical Emittance	$\varepsilon_y / \langle \eta_y^2 \rangle$	5.63×10^{-7} m^{-1}	5.90×10^{-7} m^{-1}	4.60×10^{-7} m^{-1}	4.83×10^{-7} m^{-1}
Quadrupole Vertical Alignment	$\sqrt{\langle y_{co}^2 \rangle / \langle \Delta Y^2 \rangle}$	112	115	50.9	46.0
Quadrupole Roll	$\sqrt{\langle \eta_y^2 \rangle / \langle \Delta\Theta^2 \rangle}$	86.0 m	87.0 m	7.04 m	6.04 m
Sextupole Vertical Alignment	$\sqrt{\langle \eta_y^2 \rangle / \langle \Delta Y^2 \rangle}$	309	304	52.7	64.1

We have not so far considered the vertical dispersion that arises directly from vertical steering. In a lattice with local chromatic correction, the vertical dispersion is of the same order of magnitude as the vertical closed orbit distortion. In practice, this is expected to be less than a few hundred microns. However, a lattice with long, dispersion-free straight sections, such as the TESLA or NLC damping rings, cannot have fully local chromatic correction. In this case, the ratio of the rms dispersion to the rms closed orbit distortion is of the same order of magnitude as the chromaticity. Any significant dispersion generated by steering is likely to be a limitation on the effectiveness of the correction systems studied in the simulations, since it will be difficult to achieve simultaneous correction of the dispersion and the betatron coupling. The local chromaticity is somewhat stronger in the TESLA damping ring, as a result of the proportionately larger betatron phase advance in the straight sections, compared with the NLC. Although it might be expected that the TESLA damping ring will therefore be more sensitive to closed orbit distortion, the effects are largely compensated by the fact that a global correction is applied, minimizing simultaneously the dispersion and the closed orbit distortion. This is described in more detail below.

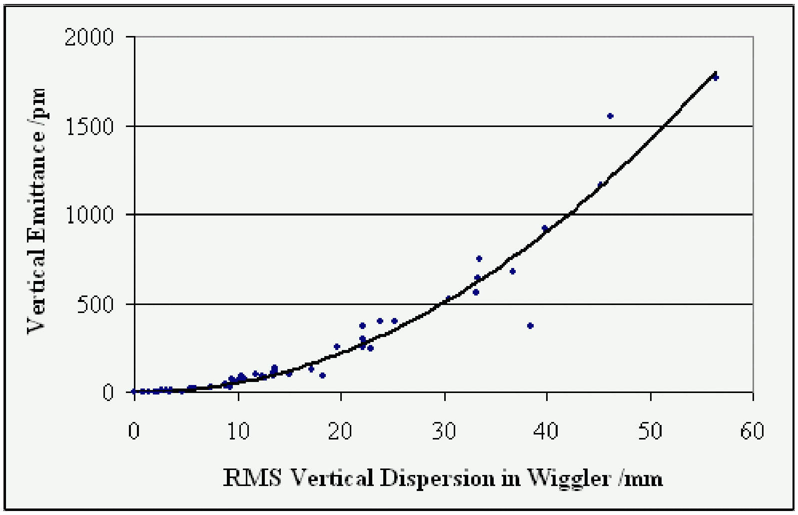

Figure 1

Vertical emittance as a function of the rms vertical dispersion in the wiggler, for vertical quadrupole misalignments in the TESLA damping ring. The solid line shows a best fit quadratic curve with coefficient 5.90×10^{-7} m^{-1}. Note that the sextupoles were turned off, so there were no sources of betatron coupling.

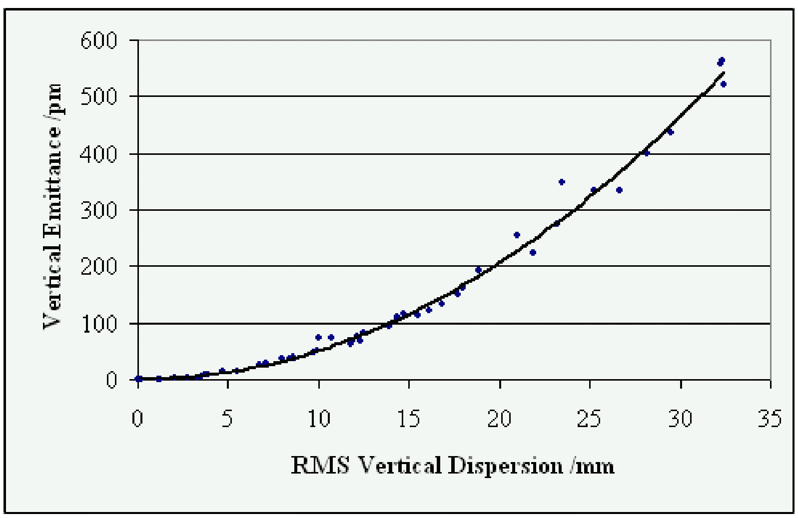

Figure 2

Vertical emittance as a function of rms vertical dispersion generated by quadrupole vertical misalignments in the NLC MDR. The solid line shows a best fit quadratic with coefficient 4.8×10^{-7} m^{-1}. Note that the sextupoles were turned off, so there were no sources of betatron coupling.

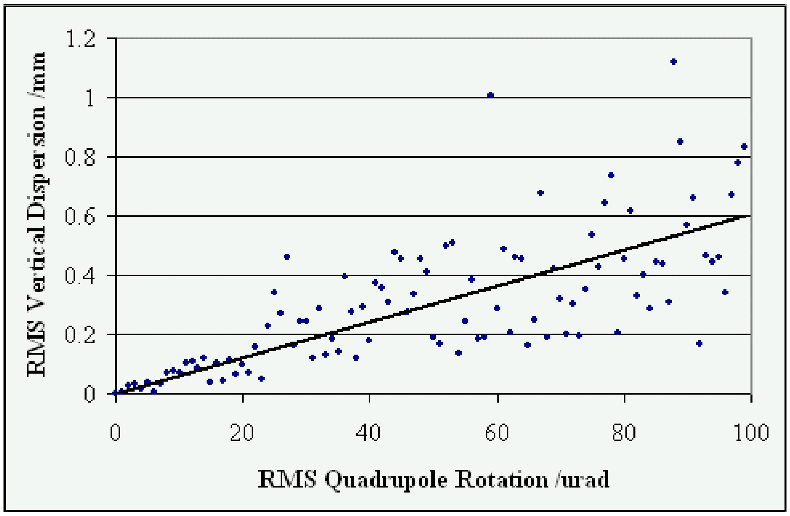

Figure 3

RMS vertical dispersion as a function of rms quadrupole rotation in the NLC MDR. The Solid line shows a best fit with gradient 6.04 m.

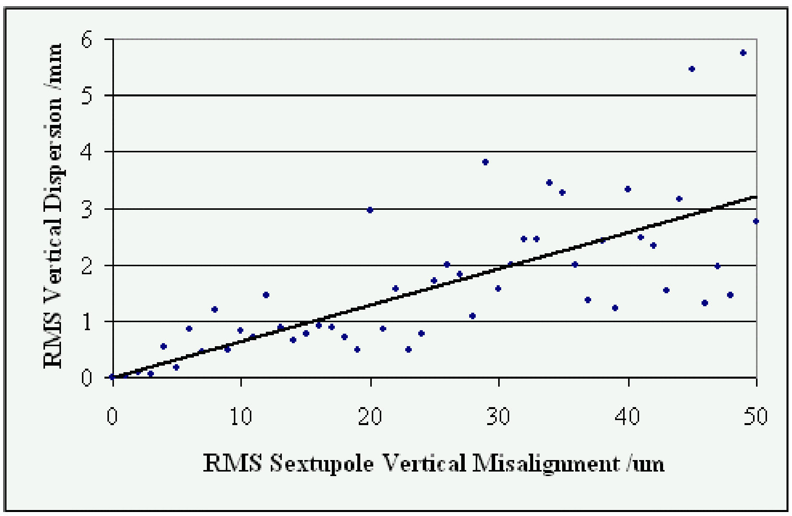

Figure 4

Correlation between rms vertical dispersion and rms sextupole vertical alignment in the NLC MDR. The solid line shows a best linear fit with coefficient 64.1.

3 Emittance Correction Simulations

The simulation codes MAD and MERLIN have been applied to the TESLA damping ring, and MERLIN has been applied to the NLC main damping ring to simulate the effects of an emittance tuning algorithm. Only a restricted subset of the errors and limitations that are to be expected in practice have so far been considered in any detail. These are:

- quadrupole vertical alignment errors;
- quadrupole rotations about the beam axis;

- sextupole vertical alignment errors;
- limited BPM resolution.

The BPM resolution limits the precision with which the dispersion may be measured. Since dispersion correction to better than 1 mm is generally required, and energy variation is limited to the order of 0.1%, the BPM resolution must be 1 µm or better. With averaging, BPMs in existing storage rings are close to achieving this resolution. Larger possible energy variations in turn relax the requirements on the BPMs or would allow for even more accurate dispersion correction.

In the simulations a BPM to quadrupole alignment of the order of 10 µm was assumed. In practice, beam-based alignment may be used to determine the vertical offset of each BPM with respect to the magnetic axis of the associated quadrupole to a precision of a few times the BPM resolution. Given that systematic effects may dominate the results of the beam-based alignment (depending on the procedures followed), this may not be a realistic assumption. Since the dispersion correction is more important than the orbit correction, and this involves taking the difference between two orbits, the BPM alignment may not be a critical issue. Nonetheless, this issue should be investigated further.

The fundamental issue we address here is whether, given only the limited subset of machine errors specified above, it might be possible to tune the damping rings to produce the specified equilibrium vertical emittance *in the low current limit*. Collective effects are beyond the scope of this note. Demonstration of effective emittance correction in simulations at the present level is a necessary but not sufficient condition for the ability of the designs to achieve the specifications. The simulations may also give some indication of the likely difficulty of achieving the specified emittances given a more complete set of machine errors, although this involves some subjective judgment.

Other considerations that will likely be important, but that have not so far been included in the simulations include:

- dipole vertical alignment and rotation errors;
- horizontal orbit and dispersion errors;
- optics errors arising from focusing variations;
- BPM rotations;
- effects of nonlinear wiggler fields;
- limitations from malfunctioning BPMs and correctors;
- tuning of the skew quadrupoles used to implement beam coupling in the TESLA damping ring.

The above issues can impact the emittance tuning in various ways, although some of them can be addressed separately. Both TESLA and NLC tuning algorithms depend on application of response matrices, which need to be accurately determined and, in the simulations, are calculated for the ideal machine. The effectiveness of the correction can be sensitive to the response matrices. Verification of storage ring optics by analysis of the measured response matrices has been applied at a number of machines and has proven a valuable technique.

Table 5

Parameters for emittance tuning simulations. The correction effectiveness is the number of seeds that were successfully corrected to within the specified emittance. Misalignments were applied with a gaussian distribution, and a cut-off of 3σ.

	NLC	TESLA
Quadrupole vertical misalignment rms	100 μm	100 μm
Quadrupole roll rms	100 μrad	100 μrad
Sextupole vertical misalignment rms	100 μm	100 μm
BPM resolution	0.5 μm	1 μm
Energy variation for dispersion measurement	±0.1%	±0.2%
Correction effectiveness [with coupling bumps]	90%	85% [70%]

3.1 TESLA Damping Ring

The TESLA correction system uses a BPM and steering magnet located at every quadrupole. Initial correction uses a combined orbit and dispersion response matrix to set the steering magnet strengths. This generally brings the vertical emittance to less than a few times the target, with further dispersion correction usually being effective in reaching the target. In the MAD simulation, the dispersion correction is achieved with a skew quadrupole located at each sextupole. This arrangement was found to be unreliable in the MERLIN simulation; substituting a magnet mover for the skew quadrupole solves the problem, and the two simulation codes then produce very similar results. Although a vertical beam offset in a sextupole has the coupling effect of a skew quadrupole, the systems are not exactly the same, since the skew quadrupole (which has fixed alignment to the design orbit in the MAD simulation, while in the MERLIN simulation it is superposed on the misaligned sextupole) has a steering effect that is not the same as a misaligned sextupole.

The errors applied to the lattice model are given in Table 5. After the initial combined orbit and dispersion correction, the residual vertical dispersion in the wiggler is generally less than 500 μm, which will generate around 0.1 pm vertical emittance. Therefore, we expect that the vertical emittance is generated principally by betatron coupling; this is confirmed by estimating the emittance using the single coupling resonance model, with knowledge of the beam offset in each sextupole. Note that in the correction algorithm, we do not measure or correct the betatron coupling directly; the required skew correction is inferred from the residual vertical dispersion. Typically, the beam offset in the sextupoles after the correction is around 400 μm rms, which is comparable to the residual closed orbit distortion. If the offsets were random, this would be expected to generate up to 50% coupling. However, a global correction is achieved for about 85% of random seeds.

As we mentioned in the previous section, the TESLA damping ring will include coupling bumps to reduce the space-charge effects, but the present studies have been limited to the lattice without the coupling bumps. It is interesting, however, to apply the vertical correction algorithm to the lattice with the coupling bumps, to give an indication of how significantly they affect the behavior of the lattice. The strong coupling in the straights suggests that both horizontal and vertical correction would be needed to achieve a properly tuned lattice, even if only vertical misalignments and were applied. In fact, we find that the correction effectiveness is reduced, but not severely, with the required vertical emittance being achieved for about 70% of random seeds.

3.2 NLC Main Damping Ring

The NLC/JLC correction system consists of BPMs placed at each quadrupole, with the quadrupoles and sextupoles positioned on movers. An orbit correction is first performed using the response matrix between the BPMs and the quadrupole movers, and a dispersion correction is then applied in an analogous fashion, using a response matrix between the vertical dispersion at the BPMs and the sextupole movers. A significant parameter for the correction is the BPM resolution, determined by the need for vertical dispersion measurement to a precision better than 1 mm, as mentioned above. In the case of the NLC, a conservative value has been chosen for the energy variation used for the dispersion measurement, which places a demanding requirement on the BPM resolution. As we mentioned above, BPMs in existing storage rings are close to achieving this resolution with averaging.

If all the vertical dispersion arises from the vertical offset of the beam in the sextupoles, then correction of the vertical dispersion (by simultaneously minimizing the vertical steering and centering the beam in the sextupoles) automatically leads to cancellation of the betatron coupling from the same source. However, vertical dispersion arises also from vertical steering and quadrupole rotations; minimization of the dispersion in the general case results in a situation where the beam has some remaining offset with respect to the sextupole centers, and some betatron coupling is expected. However, the correction algorithm generally converges towards an alignment of the quadrupoles and sextupoles on the design orbit, and the betatron coupling is small. This simple correction strategy is effective since it minimizes local errors. We find that for a given set of misalignments, the correction system is capable of meeting the specification for the vertical emittance in about 90% of cases. The residual vertical dispersion after correction is generally a few hundred microns rms, which will contribute less than 0.1 pm to the vertical emittance. The remaining vertical emittance comes from betatron coupling.

4 Conclusions

Given sets of basic alignment errors, simple algorithms based on orbit and dispersion correction are effective at correcting the vertical emittance to within the required limits for the NLC main damping rings and the TESLA damping rings. BPM resolutions of 1μm or better are required for dispersion measurements of sufficient precision, though this may be eased if larger energy variations are allowed for dispersion measurement.

The results of simple simulations aimed at determining the sensitivity of the lattices to various misalignments are in good agreement with analytical models. Both damping rings considered here are in broadly the same regime as the ATF, regarding sensitivity to misalignments.

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